

**CITY COLLEGE**  
**CITY UNIVERSITY OF NEW YORK**

**Project #1**

**HONEYCOMB SANDWICH STRUCTURE**

**ME 548: Aerostructure**

**Spring 2011**

**Prof. S.Bayer**

**Submitted By:**

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- *Objective of Experiment*

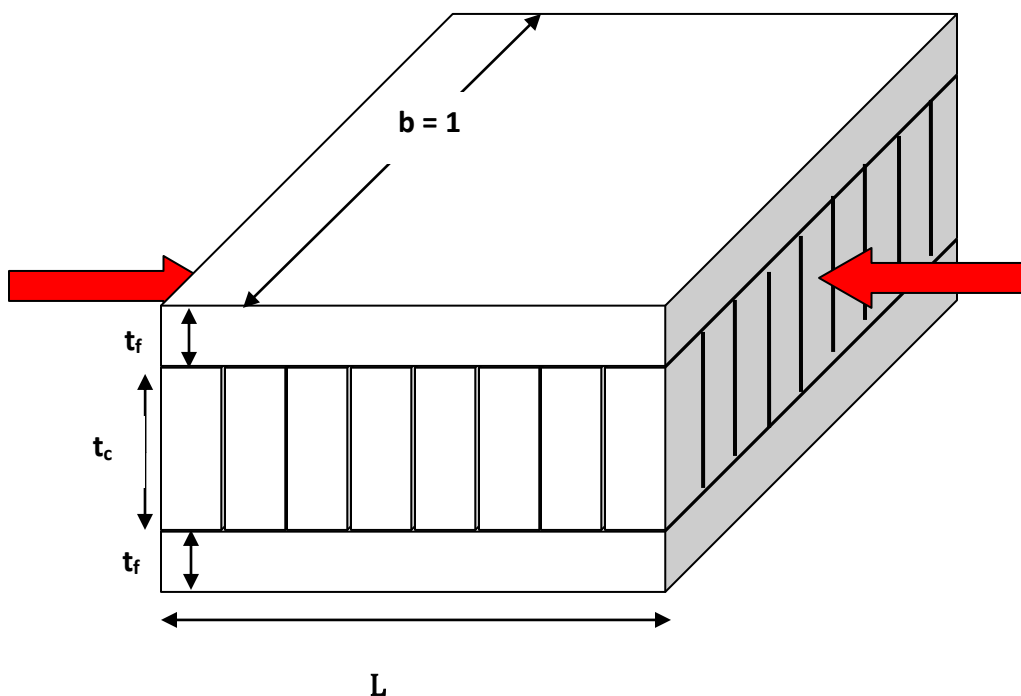
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Design a sandwich column hinged at both ends and which can sustain a buckling load  $P$ . A choice of three face material is available. Using the honeycomb construction and aluminum alloy 2024 as the core material the most cost effective design is to obtain with respect to strength and cost ratio.

- *Background*

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Innovations in aircraft design, motor vehicle technology and light-weight construction have formed the basis for the development of honeycomb structured panels. Their decisive advantage is low weight, combined with great structural strength. Because of their anti-shock properties, honeycomb structures are today used as shock-absorbent layers both in automobile construction and in sports gear and sport shoe production. They are ideally suited for design and architectural applications as a result of their optimal ratio of weight to load-bearing capacity and bending strength. In addition this composite material, which generally consists of a honeycomb core and external facing, can be adapted to individual requirements with regard to strength and choice of materials.



- *Procedure*

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$$\text{expand} \left( 2 \cdot \left( \frac{b \cdot t_f^3}{12} + b \cdot t_f \cdot \left( \frac{(t_f + t_c)^2}{2} \right) \right) \right)$$

$$\frac{2}{3} b t_f^3 + b t_f^2 t_c + \frac{1}{2} b t_f t_c^2$$

and ignoring  $t_f^3$  and  $t_f^2$  for  $t_c \gg t_f$

$$IN := \frac{1}{2} \beta \cdot t_c t_c^2$$

$$\frac{1}{2} \beta t_c^3$$

Applied Load is  $P$

$$P_{cr} := \frac{\pi^2 \cdot E \cdot IN}{L^2}$$

$$\frac{1}{2} \frac{\pi^2 E \beta t_c^3}{L^2}$$

$$t_{c \min} := \left( \frac{2 \cdot P \cdot L^2}{\pi^2 E \beta} \right)^{\frac{1}{3}}$$

$$2^{1/3} \left( \frac{P L^2}{\pi^2 E \beta} \right)^{1/3}$$

for Minimum weight  $t_c = t_{c \min}$

$$w := \rho_f \cdot \left( B \cdot L \cdot \left( \beta \cdot 2^{1/3} \left( \frac{P L^2}{\pi^2 E \beta} \right)^{1/3} + \beta \cdot 2^{1/3} \left( \frac{P L^2}{\pi^2 E \beta} \right)^{1/3} \right) \right)$$

$$+ \rho_c \cdot B \cdot L \cdot 2^{1/3} \left( \frac{P L^2}{\pi^2 E \beta} \right)^{1/3}$$

$$2 \rho_f B L \beta 2^{1/3} \left( \frac{P L^2}{\pi^2 E \beta} \right)^{1/3} + \rho_c B L 2^{1/3} \left( \frac{P L^2}{\pi^2 E \beta} \right)^{1/3}$$

$diff(w, \beta)$

$$2 \rho_f B L 2^{1/3} \left( \frac{P L^2}{\pi^2 E \beta} \right)^{1/3} - \frac{2}{3} \frac{\rho_f B L^3 2^{1/3} P}{\beta \left( \frac{P L^2}{\pi^2 E \beta} \right)^{2/3} \pi^2 E} \\ - \frac{1}{3} \frac{\rho_c B L^3 2^{1/3} P}{\left( \frac{P L^2}{\pi^2 E \beta} \right)^{2/3} \pi^2 E \beta^2}$$

$solve(\%, \beta)$

$$\frac{1}{4} \frac{\rho_c}{\rho_f}$$

$$\beta = \frac{\rho_c}{4 * (\rho_f)}$$

For the design we have core material is Aluminum 2024, and  $\rho_c = 0.1 \frac{lb}{in^3}$

For facing material we have

	Facing Material	Stress, Kips/in <sup>2</sup>	Density, lb/in <sup>3</sup>	Young's Modulus, Kips/in <sup>2</sup>	Relative Price Per Pound, \$
1	Aluminum 2024	66	0.1	10.5	10
2	Magnesium Alloy	40	0.065	0.5	15
3	Laminated Composite	30	0.050	2.5	20

**L= 48 inch and P=6000 lb**

Young's Modulus of Composite is given by:

$$E_{comp} = E_{core} * \vartheta_{film} + E_{film} * \vartheta_{core}$$

where,

$$E_{comp} = \text{Young's Modulus of Elasticity of Composite} \left( \frac{lb}{in^2} \right)$$

$$E_{core} = \text{Young's Modulus of Elasticity of Core} \left( \frac{lb}{in^2} \right)$$

$$E_{film} = \text{Young's Modulus of Elasticity of film} \left( \frac{lb}{in^2} \right)$$

$$\vartheta_{film} = \text{volume fraction of film} = \frac{V_{film}}{V_{total}}$$

$$\vartheta_{core} = \text{volume fraction of core} = \frac{V_{core}}{V_{total}}$$

$$E_{comp} = E_{core} * \frac{V_{film}}{V_{total}} + E_{film} * \frac{V_{core}}{V_{total}}$$

$$E_{comp} = E_{core} * \frac{(2 * t_f * B * L)}{(B * L * (2 * t_f + t_c))} + E_{film} * \frac{(t_c * B * L)}{(B * L * (2 * t_f + t_c))}$$

$$E_{comp} = E_{core} * \frac{2 * t_f}{(2 * t_f + t_c)} + E_{film} * \frac{t_c}{(2 * t_f + t_c)}$$

$$\text{Similarly, Ultimate stress, } \sigma \left( \frac{lb}{in^2} \right)$$

$$\rho_{comp} = \rho_{core} * \frac{2 * t_f}{(2 * t_f + t_c)} + \rho_{film} * \frac{t_c}{(2 * t_f + t_c)}$$

$$\text{Similarly, Ultimate stress, } \sigma \left( \frac{lb}{in^2} \right)$$

$$\sigma_{comp} = \sigma_{core} * \frac{2 * t_f}{(2 * t_f + t_c)} + \sigma_{film} * \frac{t_c}{(2 * t_f + t_c)}$$

For valid design sustainability:

*For a long section of member of length L, and under applied load P*

$$P < P_{cr} \text{ and } \sigma_{comp} < \sigma_{cr}$$

where

$$P_{cr} = \frac{\pi^2 E_{comp} * I_{comp}}{L_{eff}^2}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E_{comp} * I_{comp}}{L_{eff}^2 * B * (2 * t_f + t_c)}$$

where

$L_{eff} = L =$  length of the member, for force applied from two end, 48 inch

$B =$  width of of the member, 1 inch

$P =$  Applied Load = 6000 lb

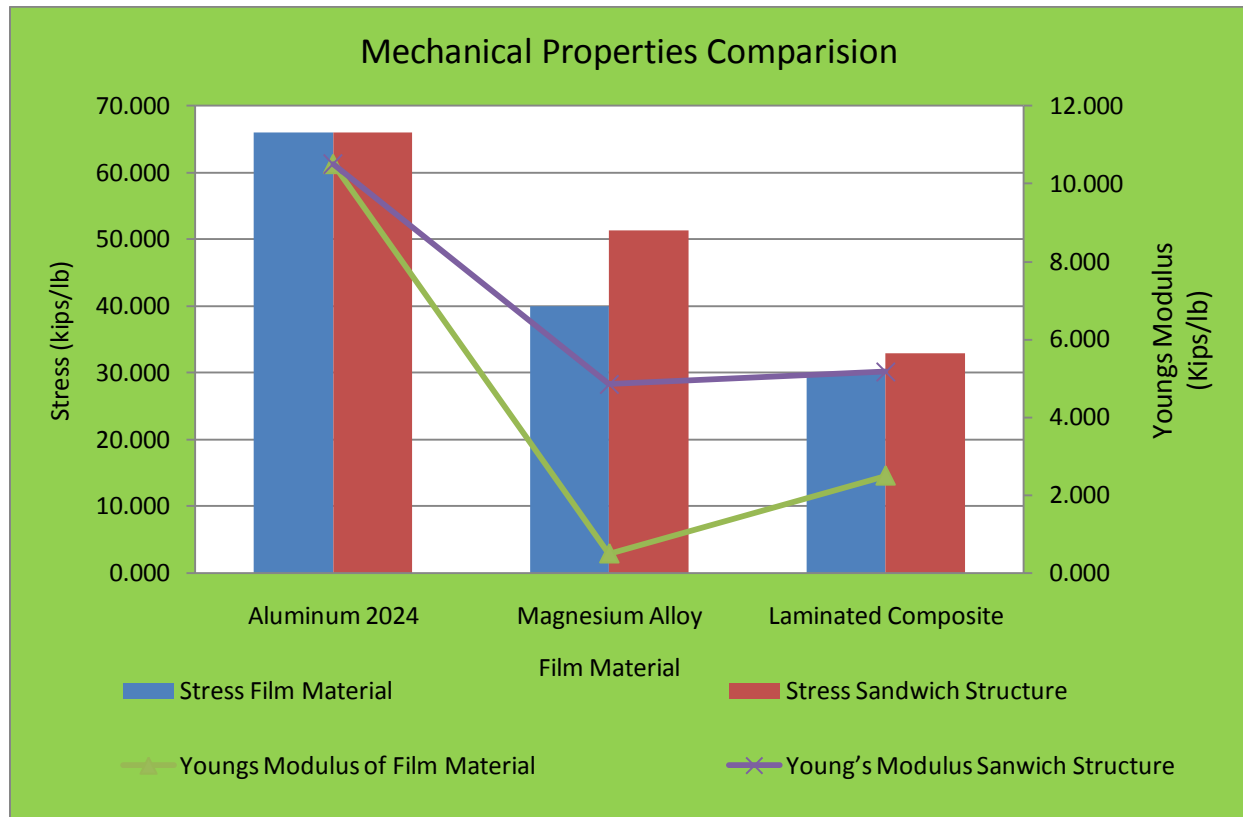
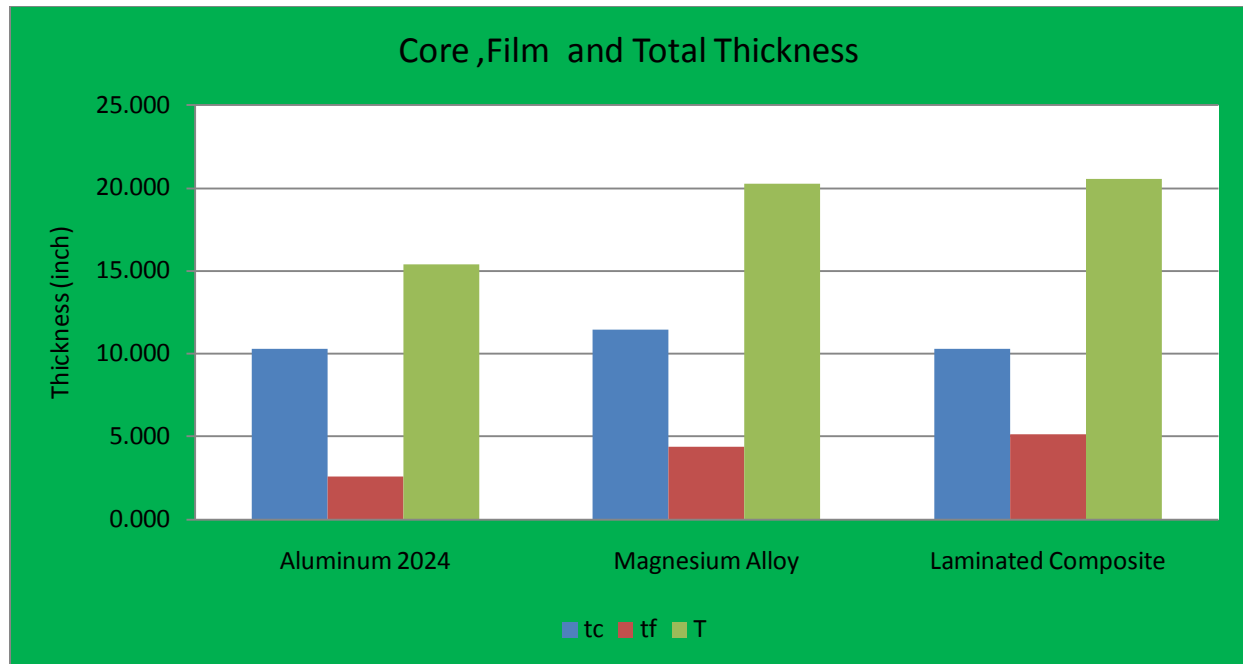
$$I_{comp} = 2 * \left( \frac{B * t_f^3}{12} + B * t_f * \left( \frac{t_f + t_c}{2} \right)^2 \right)$$

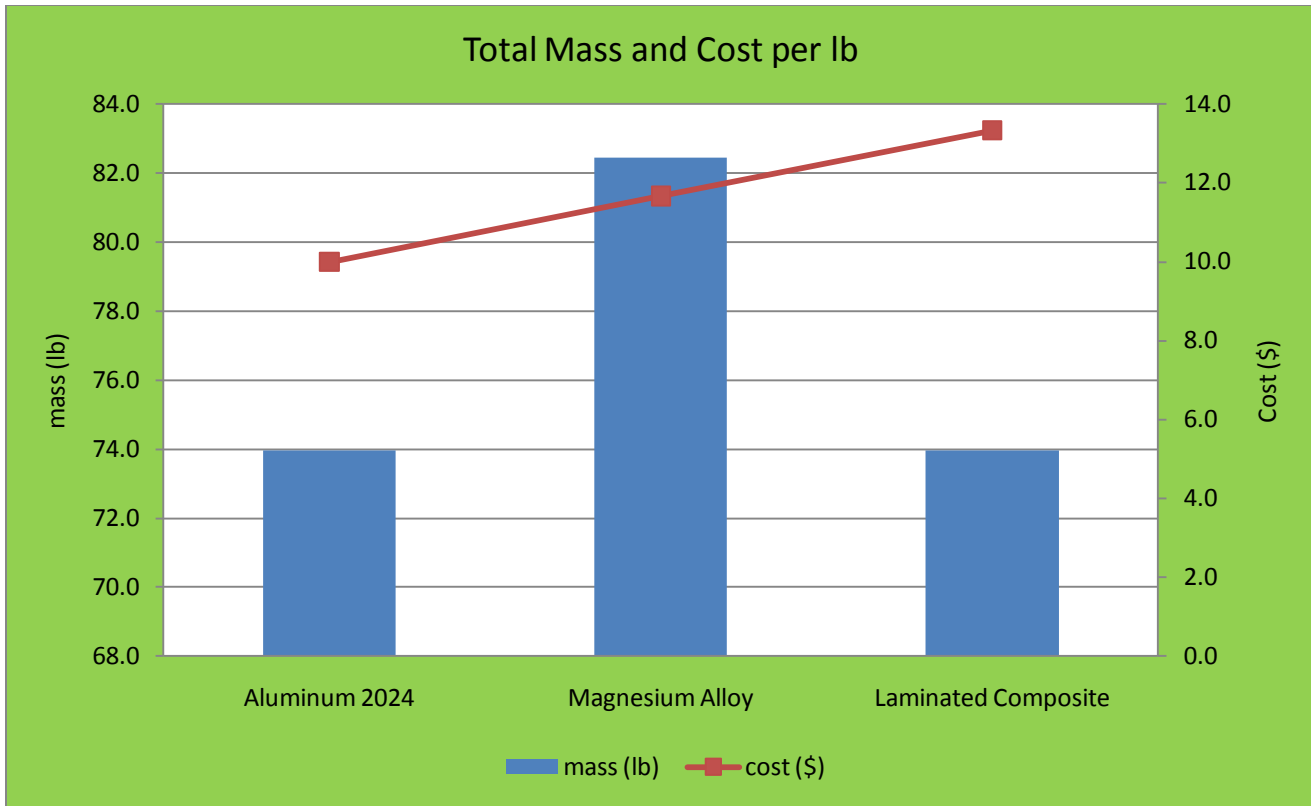
Facing Material	Core: Aluminum Alloy 2024		
	$t_f, inch$	$t_c, inch$	$\beta$
Aluminum 2024	2.555	10.219	0.250
Magnesium Alloy	4.405	11.453	0.385
Laminated Composite	5.137	10.274	0.500

Composite Material Properties						
<i>Stress, kips in<sup>2</sup></i>	<i>Density, lb in<sup>3</sup></i>	<i>Young's Modulus, kip in<sup>2</sup></i>	<i>Mass, lb</i>	<i>Relative Price Per Pound, \$</i>	<i>Critical Load, lb</i>	<i>Critical Stress, lb in<sup>2</sup></i>
66.000	0.100	10.500	73.973	10.000	6000.486	389.364
51.304	0.080	4.848	82.464	11.667	6000.000	296.099
33.000	0.075	5.167	73.971	13.333	6000.000	292.007

- Results

### Aluminum 2024 Core





## ● Conclusion

Therefore from the analysis, Aluminum 2024 is the most cost effective with respect to buckling load however, this difference is compromise able with the weight ration which with weight optimization with respect to the same nominal stress Laminated Composite is most effective composite material as respect to mass to cost ratio. Though we assumed that the thickness of the film is very less than thickness of core, however in the graph the ratio of  $\frac{t_f}{t_c} = .25$  for laminated composite and  $\frac{t_f}{t_c} = 0.5$ , therefore validating the design as per the assumption the winner is ***Aluminum 2024 core with Aluminum 2024 as film.***



## • Appendix

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$A_c := A_f$	$A_f$
$L := 48$	48
$B := 1$	1
$\rho_{core} := .1$	0.1
$\sigma_{core} := 66 \cdot 10^3$	66000
$Al\rho_f := .1$	0.1
$Mg\rho_f := .065$	0.065
$La\rho_f := .050$	0.050
$E_c := 10.5 \cdot 10^3$	10500.0
$AlE_f := 10.5 \cdot 10^3$	10500.0
$MgE_f := 0.5 \cdot 10^3$	500.0
$LaE_f := 2.5 \cdot 10^3$	2500.0
$Al\sigma_f := 66 \cdot 10^3$	66000
$Mg\sigma_f := 40 \cdot 10^3$	40000
$La\sigma_f := 30 \cdot 10^3$	30000
$P := 6000$	6000
$cost_{core} := 10$	

	10
$Alcost_{film} := 10$	
	10
$Mgcost_{film} := 15$	
	15
$Lacost_{film} := 20$	
	20
$Al\beta := \frac{1}{4} \frac{\rho_{core}}{Al\rho_f}$	
	0.2500000000
$Mg\beta := \frac{1}{4} \frac{\rho_{core}}{Mg\rho_f}$	
	0.3846153846
$La\beta := \frac{1}{4} \frac{\rho_{core}}{La\rho_f}$	
	0.5000000000
$Alt_f := Al\beta \cdot t_{c_{min}}$	
	0.2500000000 $t_{c_{min}}$
$Mgt_f := Mg\beta \cdot t_{c_{min}}$	
	0.3846153846 $t_{c_{min}}$
$Lat_f := La\beta \cdot t_{c_{min}}$	
	0.5000000000 $t_{c_{min}}$
$AlINERTIA := \frac{1}{2} Alt_f \cdot t_{c_{min}}^2$	
	0.1250000000 $t_{c_{min}}^3$
$MgINERTIA := \frac{1}{2} Mgt_f \cdot t_{c_{min}}^2$	
	0.1923076923 $t_{c_{min}}^3$
$LaINERTIA := \frac{1}{2} Lat_f \cdot t_{c_{min}}^2$	
	0.2500000000 $t_{c_{min}}^3$
$V_{core} := \left( B \cdot L \cdot \left( t_{c_{min}} \right) \right)$	
	48 $t_{c_{min}}$

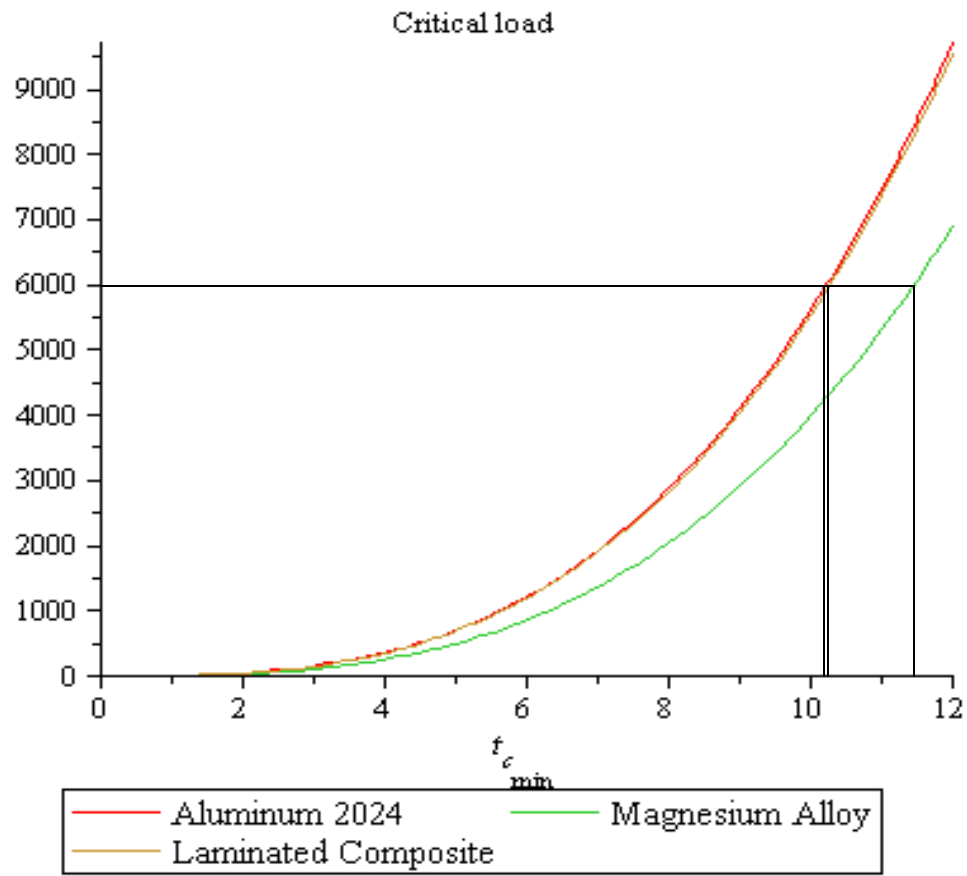
$AlV_{film} := (B \cdot L \cdot (Alt_f + Alt_f))$	$24.00000000t_{c_{min}}$
$AlV_{total} := (B \cdot L \cdot (Alt_f + Alt_f + t_{c_{min}}))$	$72.00000000t_{c_{min}}$
$MgV_{film} := (B \cdot L \cdot (Mgt_f + Mgt_f))$	$36.92307692t_{c_{min}}$
$MgV_{total} := (B \cdot L \cdot (Mgt_f + Mgt_f + t_{c_{min}}))$	$84.92307691t_{c_{min}}$
$LaV_{film} := (B \cdot L \cdot (Lat_f + Lat_f))$	$48.00000000t_{c_{min}}$
$LaV_{total} := (B \cdot L \cdot (Lat_f + Lat_f + t_{c_{min}}))$	$96.00000000t_{c_{min}}$
$Alv_f := \frac{AlV_{film}}{AlV_{total}}$	$0.3333333333'$
$Alv_c := \frac{V_{core}}{AlV_{total}}$	$0.6666666666'$
$AlE_{comp} := AlE_f \cdot Alv_c + E_c \cdot Alv_f$	$10500.00000'$
$AlP_{cr} := \frac{\pi^2 \cdot AlE_{comp} \cdot AlINERTIA}{L^2}$	$0.5696614583\pi^2 t_{c_{min}}^3$
$Mgv_f := \frac{MgV_{film}}{MgV_{total}}$	$0.434782608'$
$Mgv_c := \frac{V_{core}}{MgV_{total}}$	$0.565217391'$
$MgE_{comp} := MgE_f \cdot Mgv_c + E_c \cdot Mgv_f$	$4847.82608'$
$MgP_{cr} := \frac{\pi^2 \cdot MgE_{comp} \cdot MgINERTIA}{L^2}$	$0.4046329197\pi^2 t_{c_{min}}^3$

$LaV_f := \frac{LaV_{film}}{LaV_{total}}$	0.5000000000
$LaV_c := \frac{V_{core}}{LaV_{total}}$	0.5000000000
$LaE_{comp} := LaE_f \cdot Alv_c + E_c \cdot Alv_j$	5166.666667
$LaP_{cr} := \frac{\pi^2 \cdot LaE_{comp} \cdot LaINERTIA}{L^2}$	$0.5606192131 \pi^2 t_{c \min}^3$
$Al\rho_{comp} := Al\rho_f \cdot Alv_c + \rho_{core} \cdot Alv_j$	0.1000000000
$Mg\rho_{comp} := Mg\rho_f \cdot MgV_c + \rho_{core} \cdot MgV_j$	0.0802173913
$La\rho_{comp} := La\rho_f \cdot LaV_c + \rho_{core} \cdot LaV_j$	0.0750000000
$Al\sigma_{comp} := Al\sigma_f \cdot Alv_c + \sigma_{core} \cdot Alv_j$	66000.00000
$Mg\sigma_{comp} := Mg\sigma_f \cdot MgV_c + \sigma_{core} \cdot MgV_j$	51304.34781
$La\sigma_{comp} := La\rho_f \cdot LaV_c + \sigma_{core} \cdot LaV_j$	33000.02500

```

plot([AlPcr, MgPcr, LaPcr], tc min = 0..12, title = "Critical load," legend
= ["Aluminum 2024","Magnesium Alloy","Laminated Composite"])

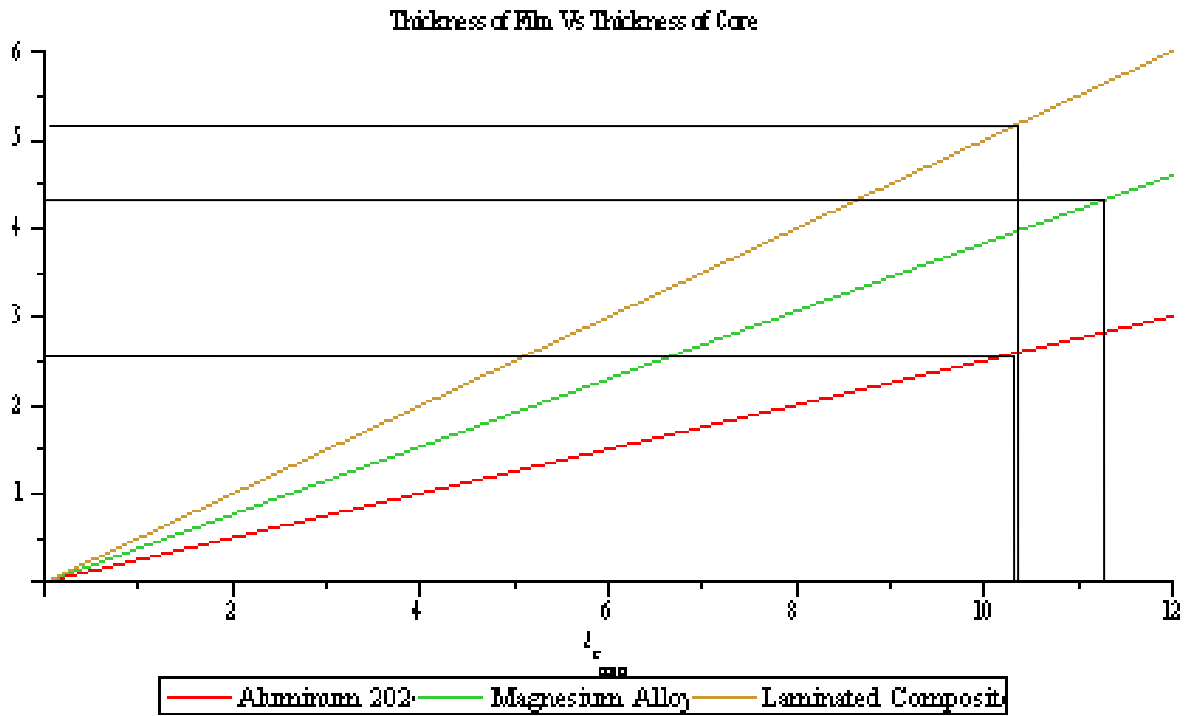
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plot([Alt_f, Mgt_f, Lat_f], t_c_min = 0..12, title
    = "Thickness of Film Vs Thickness of Core", legend
    = ["Aluminum 2024", "Magnesium Alloy", "Laminated Composite"])

```



$$Alcost := B \cdot L \cdot (2 \cdot Alt_f) \cdot Al\rho_f \cdot Alcost_{film} + B \cdot L \cdot \left( t_{c_{min}} \right) \cdot \rho_{core} \cdot cost_{core}$$

$72.00000000 t_{c_{min}}$

$$Mgcost := B \cdot L \cdot (2 \cdot Mgt_f) \cdot Mg\rho_f \cdot Mgcost_{film} + B \cdot L \cdot \left( t_{c_{min}} \right) \cdot \rho_{core} \cdot cost_{core}$$

$84.00000000 t_{c_{min}}$

$$Lacost := B \cdot L \cdot (2 \cdot Lat_f) \cdot La\rho_f \cdot Lacost_{film} + B \cdot L \cdot \left( t_{c_{min}} \right) \cdot \rho_{core} \cdot cost_{core}$$

$96.00000000 t_{c_{min}}$

plot([Alcost, Mgcost, Lacost],  $t_{c_{min}} = 0 \dots 12$ , title  
= "Cost Comparision", legend = ["Aluminum 2024"  
"Magnesium Alloy", "Laminated Composite"])

